第２回構造物の破壊過程解明に基づく
地震防災性向上に関するシンポジウム論文集　2001年3月

NUMERICAL ANALYSIS OF MULTIPLE CRACKS IN CONCRETE USING THE DISCRETE MODELING APPROACH

Zihai SHI[1], Masayasu OHTSU[2], Masaki SUZUKI[3] and Yoshihisa HIBINO[4]

[1] Dr. Eng., Research and Development Center, Nippon Koei Co., Ltd.,
(2304 Inarihara, Kukizaki-machi, Inashiki-gun, Ibaraki 300-1259, Japan)
[2] Dr. Eng., Professor, Graduate School of Science and Technology, Kumamoto University,
(Kurokami 2-39-1, Kumamoto 860-8555, Japan)
[3] Researcher, Research and Development Center, Nippon Koei Co., Ltd.,
(2304 Inarihara, Kukizaki-machi, Inashiki-gun, Ibaraki 300-1259, Japan
[4] Assistant Manager, Construction Engineering Center, Tokyo Electric Power Company,
(1-3 Uchisaiwai-cho 1-chome, Chiyoda-ku, Japan)

本研究では、一本のひび割れの数値解析によく用いられている離散ひび割れモデルを、ひび割れ先端を制御する手法に基づいて、複数のひび割れの数値解析へ拡張した。本モデルでは、任意の一本のひび割れを進展させるのに必要な外力をそれ以外のひび割れの進展を拘束する中で算出し、その最小値に基づいてひび割れの進展順を決定する。この過程の中で、拘束されているひび割れの進展位置を修正し、無効となる数値解を除去することによって、様々なひび割れ進展パターンを形成することができる。本モデルの有効性は、実規模のトンネル覆工破壊実験との検証によって確認できた。

Key Words : *multiple cracks, discrete modeling, fictitious crack, minimum load criterion*

1. INTRODUCTION

In many engineering applications involving an aging or partially damaged concrete structure, the available information on structural integrity is often confined to the crack-opening widths of several distinct cracks in the structure. Take the concrete lining of an aging waterway tunnel, one of the major constituents of a hydraulic power facility, as an example. Due to the formation of caves behind the concrete lining in the ceiling area, structural deformation progresses under external compression, leading to the formation of several distinct longitudinal cracks, as shown in **Fig. 1**. Obviously, for evaluating the structural safety of these aging waterway tunnels, crack-opening widths serve as an important index. In these circumstances the commonly adopted smeared crack approach for multiple cracks is inapplicable because of its continuum assumption for the cracked concrete, and hence numerical analysis of multiple cracks using discrete modeling techniques is desirable.

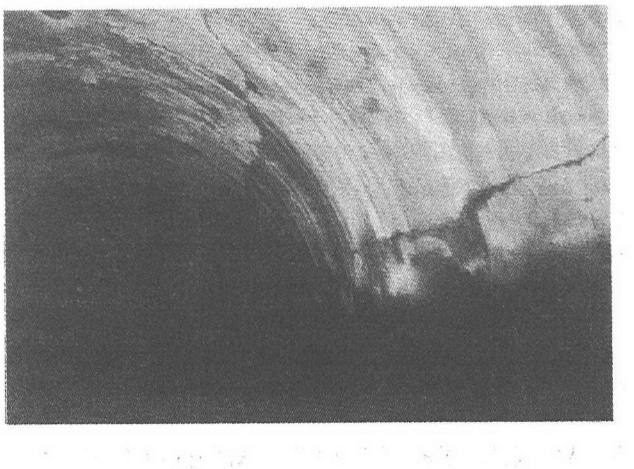
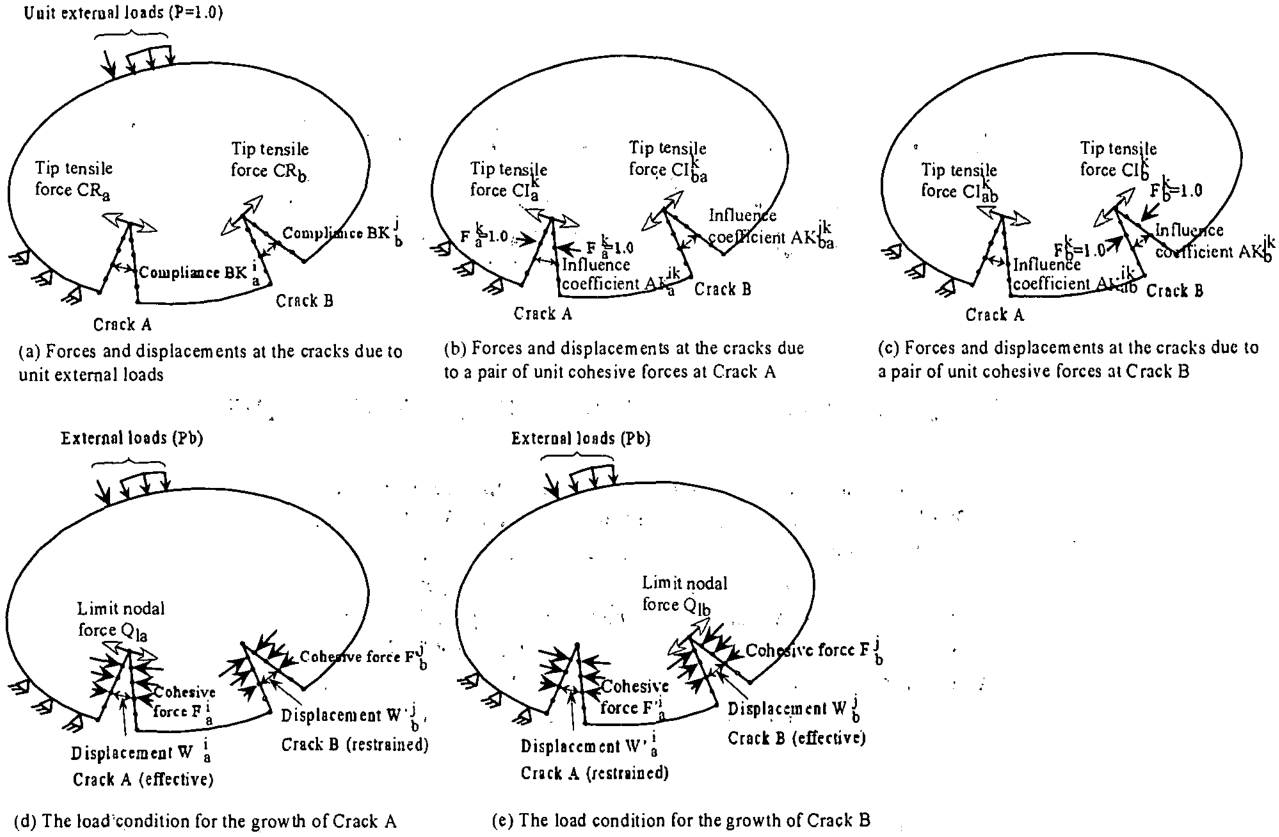


Fig. 1 An aging waterway tunnel with large longitudinal cracks



(a) Forces and displacements at the cracks due to unit external loads

(b) Forces and displacements at the cracks due to a pair of unit cohesive forces at Crack A

(c) Forces and displacements at the cracks due to a pair of unit cohesive forces at Crack B

(d) The load condition for the growth of Crack A

(e) The load condition for the growth of Crack B

Fig. 2 The concept of the crack-tip-controlled modeling of multiple cracks

In the present study, the discrete crack approach, which has been extensively used for analyzing single cracks[1),2),3),4),5)], is extended for numerical analysis of multiple cracks, based on a fictitious crack model and the crack-tip-controlled numerical algorithm. An explicit formulation of the mode I-type crack propagation for multiple cracks is presented, and a minimum load criterion for crack extension is proposed for its numerical implementation. Other principles necessary for analyzing multiple-crack problems are also explained. Next, the newly developed finite element program for discrete modeling of multiple cracks is used for analyzing the failure process of a real-size tunnel specimen under distributed load conditions, and the numerical results are found to be in good agreement with available documented experimental results.

2. DISCRETE CRACK MODELING OF MULTIPLE CRACKS

The primary obstacle in applying the analytical procedure of the crack-tip-controlled approach to problems involving multiple cracks lies in the ambiguity of determining the effective crack or cracks among a group of potentially active cracks in numerical analysis. To overcome this difficulty, the strategy employed here is to take turn to assume each crack as an effective crack while restraining the growth of others, and calculate the load required for propagating this particular crack by solving the relevant crack equations. The true effective crack, as well as the force and displacement fields, is then determined based on a minimum load criterion which stipulates the propagation of the effective crack at the minimum load. When an invalid solution is encountered, manifested either by the tip tensile stress exceeding the tensile strength or by overlapping of the crack surfaces (with negative crack opening displacements obtained) at restrained cracks, the configuration of the corresponding crack is then modified accordingly by releasing or closing the tip dual nodes. Then the case is recalculated. Eventually, this may result in other possible crack propagation patterns besides the initially assumed single crack growth, which include simultaneous crack growth involving several cracks, and crack growth accompanied by crack closure.

Fig. 2 illustrates two prescribed cracks of the mode I-type, crack A and crack B, and the crack equations that govern their propagation are derived

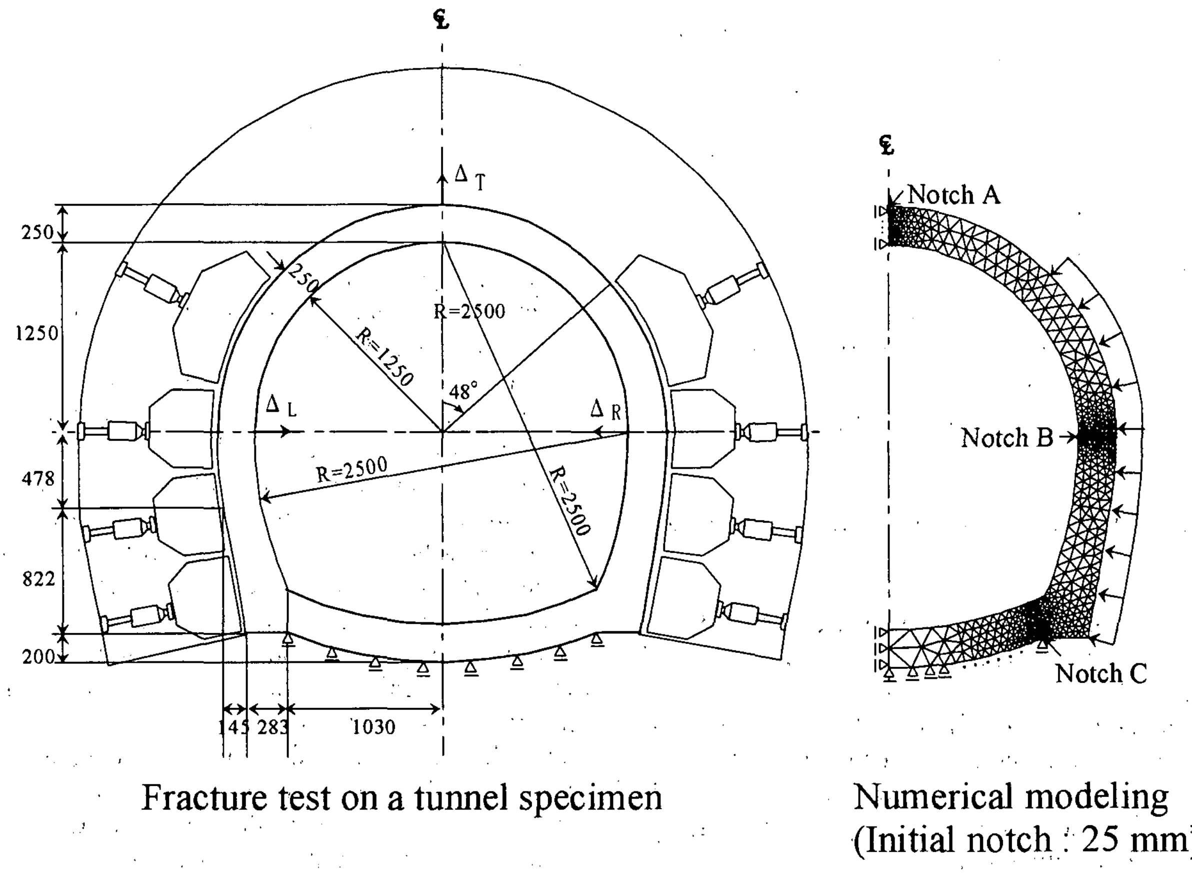


Fig. 3 A fracture test of a tunnel specimen and the numerical modeling

below. For clarity, the forces and displacements of the restrained crack are denoted using apostrophe marks.

To begin with, crack A is assumed to be the effective crack. Along each fictitious crack the cohesive forces and displacements follow the strain-softening law of concrete:

$$F_a^i = f(W_a^i) \tag{1}$$

$$F_b'^j = f(W_b'^j) \tag{2}$$

where i = 1,..., N ; j = 1,..., M, and N and M are the number of nodes inside each fictitious crack, respectively.

For crack A to extend, the tensile force at its tip must reach the limit value, i.e.,

$$Q_{la} = CR_a \cdot P_a + \sum_{i=1}^{N} CI_a^i F_a^i + \sum_{j=1}^{M} CI_{ab}^j F_b'^j \tag{3}$$

where Q_{la} is the limit nodal force calculated from the tensile strength of concrete. Note that the tensile forces at the tip of crack A, CR_a, CI_a^i and CI_{ab}^j, are due to a unit external load, a pair of unit cohesive forces at the *i*th node of crack A, and a pair of unit cohesive forces at the *j*th node of crack B, respectively. The external load P_a stands for the load required for the propagation of crack A, while the growth of crack B is restrained.

The displacements along the two fictitious cracks are given by

$$W_a^i = BK_a^i \cdot P_a + \sum_{k=1}^{N} AK_a^{ik} F_a^k + \sum_{j=1}^{M} AK_{ab}^{ij} F_b'^j \tag{4}$$

$$W_b'^j = BK_b^j \cdot P_a + \sum_{i=1}^{N} AK_{ba}^{ji} F_a^i + \sum_{k=1}^{M} AK_b^{jk} F_b'^k \tag{5}$$

Here, the compliances BK_a^i at crack A and BK_b^j at crack B are due to the external load P_a. The influence coefficients AK_a^{ik} and AK_{ab}^{ij} are the displacements at the *i*th node of crack A due to a pair of unit cohesive forces at the *k*th node of crack A, and a pair of unit cohesive forces at the *j*th node of crack B, respectively. Similarly, the influence coefficients AK_{ba}^{ji} and AK_b^{jk} give the displacements at the *j*th node of crack B due to a pair of unit

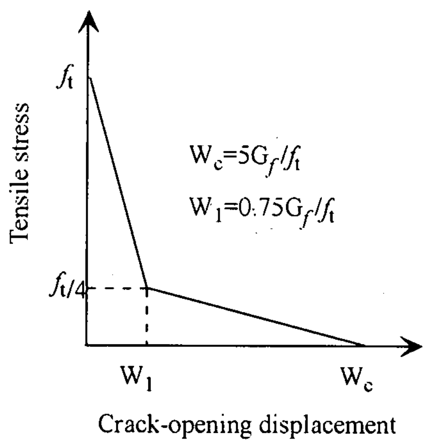


Fig. 4 The bi-linear strain-softening relation of concrete

cohesive forces at the ith node of crack A, and a pair of unit cohesive forces at the kth node of crack B, respectively.

Equations (1) to (5) form the so called crack equations with the number of equations (2N + 2M + 1) matching the number of unknowns (2N + 2M + 1), and thus, the growth of the prescribed multiple cracks is uniquely defined.

Alternatively when crack B is the assumed effective crack, the crack equations are derived and given by

$$F_a^{\prime i} = f(W_a^{\prime i}) \quad (6)$$

$$F_b^{j} = f(W_b^{j}) \quad (7)$$

$$Q_{lb} = CR_b \cdot P_b + \sum_{i=1}^{N} CI_{ba}^{i} F_a^{\prime i} + \sum_{j=1}^{M} CI_b^{j} F_b^{j} \quad (8)$$

$$W_a^{\prime i} = BK_a^{i} \cdot P_b + \sum_{k=1}^{N} AK_a^{ik} F_a^{\prime k} + \sum_{j=1}^{M} AK_{ab}^{ij} F_b^{j} \quad (9)$$

$$W_b^{j} = BK_b^{j} \cdot P_b + \sum_{i=1}^{N} AK_{ba}^{ji} F_a^{\prime i} + \sum_{k=1}^{M} AK_b^{jk} F_b^{k} \quad (10)$$

Here, the external load P_b is the required load for the propagation of crack B, while the growth of crack A is restrained.

Equations (1) to (10) form the crack equations for discrete modeling of two cracks. Apparently, the above equations can be readily extended to include any number of cracks. For brevity of presentation, the crack equations on arbitrary number of cracks are omitted here.

As previously stated, numerical implementation of the above crack equations is based on the minimum load criterion for determining the true

Table 1 Material properties of the tunnel specimen

E (GPa)	ν	f_c (MPa)	f_t (MPa)	G_f (N/mm)	$W_c(=5G_f/f_t)$ (mm)
20.00	0.20	20.00	2.00	0.10	0.25

effective crack. Note that the solutions of the crack equations are checked for negative crack opening displacements (COD) and the reversing or superficial increasing of the cohesive stresses during the closing of a restrained crack. These invalid solutions are corrected either by resetting the tip of the corresponding fictitious crack backward and recalculating the case, or simply by reducing the increased cohesive stresses to the previous level to ensure the decrease of concrete rigidity is an irreversible process. Again, following the stress analysis, the tip tensile stresses at restrained cracks are checked against the tensile strength of concrete. If a tip tensile stress is found to exceed the tensile strength, the corresponding tip dual nodes are then disconnected to propagate the crack, and the case is recalculated. Because of the space limitation, the flow chart showing the numerical procedures for analyzing multiple cracks is omitted here.

3. NUMERICAL STUDIES ON TUNNEL SPECIMENS WITH MULTIPLE CRACKS

A fracture test of a real-size concrete lining specimen of a waterway tunnel under distributed loads applied to the side walls, as previously documented[6], is shown in **Fig. 3**. The test was carried out to investigate the failure process of a tunnel with caving above the ceiling area, and to study the remaining load-carrying capacity after the formation of cracks. The numerical case to be studied is also shown in **Fig. 3**, which is a half model of the test specimen, taking into account the assumed symmetric conditions of the experiment. As illustrated, the numerical case contains three initial notches whose positions are determined roughly from the crack patterns of the test results. The material properties of the test specimen are summarized in **Table 1**. Note that the bi-linear strain softening relation in **Fig. 4** is employed to solve the crack equations.

Although no measurements of the CODs were taken during the test, the crack propagation patterns were carefully recorded. The test and numerical results are shown in **Fig. 5**. It was reported that

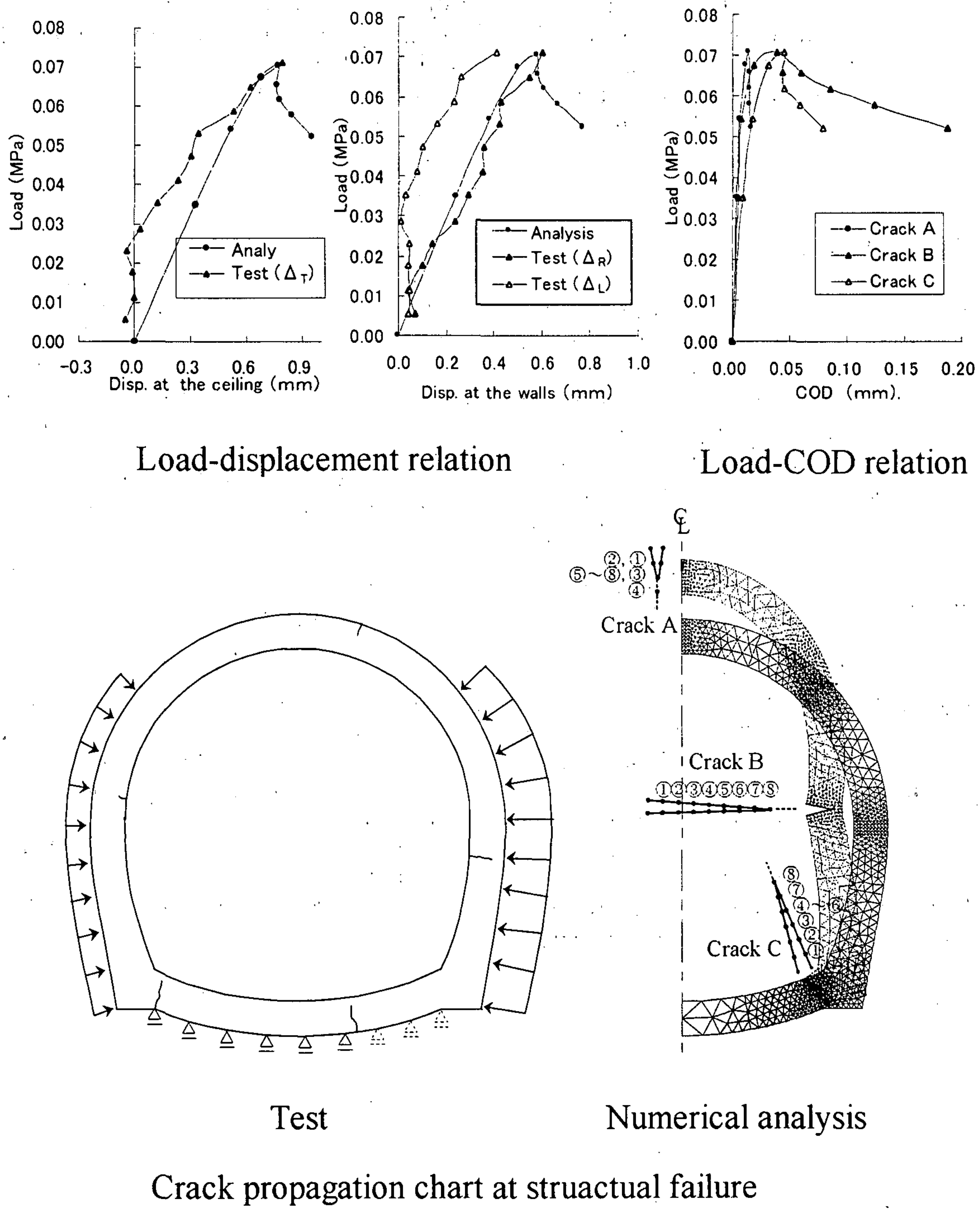


Fig. 5 Numerical results of the crack analysis of the tunnel specimen

during the experiment a certain degree of eccentric loading occurred, generating a higher pressure-load on the right wall. This is evident from the load-displacement relations separately measured on the left and right walls, as well as from the crack propagation chart of the experiment. Despite this discrepancy in the loading conditions between the actual test and the numerical study, judging from the load-displacement relations it is still reasonable to conclude that the present numerical model reproduces well the general structural response of the tunnel specimen. Regarding the crack behaviors, crack B in the middle of the wall and crack C in the bottom plate are found to be most active, contrasting sharply to the slow opening of crack A in the ceiling area. The growth of crack A stops as the maximum load is obtained at the fourth step of the computation, and remains inactive till structural failure. Note that the maximum COD of crack B reaches roughly 0.2 mm. Taking into consideration the actual eccentric load conditions in the test, the crack behaviors and crack propagation patterns obtained from the numerical analyses are regarded as in good agreement with the experimental observations.

4. CONCLUSIONS

The discrete crack approach is extended for numerical analysis of multiple cracks, based on the fictitious crack model and the crack-tip-controlled numerical algorithm. Assuming each crack as an effective crack while restraining the growth of others in turn, an explicit formulation of mode I-type crack propagation for multiple cracks is presented. The true effective crack is then determined based on the minimum load criterion which stipulates the propagation of the effective crack at the minimum load. By eliminating invalid solutions to ensure proper crack behavior, this numerical model is capable of producing a variety of crack propagation patterns.

Although numerical analysis of multiple cracks using the discrete approach is deemed difficult, the proposed method presents a systematic solution to the problem, based on simple and clear principles of the fracture mechanics in concrete. The accuracy of the model is demonstrated through solving a structural problem containing multiple cracks, i.e., the fracture tests of a real-size tunnel specimen. Comparisons are made between the numerical analyses and the experimental results, with an emphasis given to the load-displacement relations and the crack propagation patterns. The agreement between the two results is found to be excellent.

REFERENCES

1) Hillerborg, A., Modeer, M., and Peterson, P. E.: Analysis of crack formation and crack growth in concrete by means of fracture mechanics and finite elements, *Cement and concrete research*, Vol. 6, pp. 773-782, 1976.
2) Hillerborg, A.: Numerical method to simulate softening and fracture of concrete, *Fracture mechanics of concrete*, G. C. Sih and A. DiTommaso, eds., Martinus Nijhoff Publishers, pp. 141-170, 1985.
3) Carpinteri, A., Valente, S., Ferrara, G. and Imperato, L.: Experimental and numerical fracture modeling of a gravity dam, *Fracture mechanics of concrete structures*, Z. P. Bazant ed., Elsevier Applied Science, London and New York, pp. 351-360, 1992.
4) Ohtsu, M.: Tension softening properties in numerical analysis, *Colloquium on fracture mechanics of concrete structures*, JCI Committee report, pp. 55-65, 1990.
5) Ohtsu, M. and Chahrour, A. H.: Fracture analysis of concrete based on the discrete crack model by the boundary element method, *Fracture of brittle disordered materials: concrete, rock and ceramics*, G. Baker and B. L. Karihaloo, eds., E & FN Spon, London, pp. 55-65, 1993.
6) Abo, H., Tanaka, M. and Yoshida, N.: Development of a maintenance system for waterway tunnels, *Electric Power Civil Engineering*, JEPOC, No. 287, pp. 42-46, 2000.