

# NONLINEAR DYNAMIC ANALYSIS OF PILE FOUNDATION

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A new methodology to incorporate the nonlinear behavior of soil, in dynamic analysis of piles is proposed. Approach is based on Green's function formulation for the linear pile analysis while equivalent linearization is used to include the material nonlinearity of the soil media. Hyperbolic model is used to define the nonlinear stress-strain relationship of the soil. Results are derived for seismic response as well as impedance functions for a single pile and pile group. It is found that both impedance functions and response are greatly affected by the nonlinear behavior of soil media.

**Key Words:** *Pile Foundation, Dynamics, Nonlinear Effects, Hyperbolic Model, Seismic Response, Impedance Functions.*

## 1. INTRODUCTION

Dynamic response of a pile group greatly affects the response of the structures supported by them. Thus proper analysis of structure-pile-soil system requires adequate analysis of underneath soil-pile system. Good amount of literature is available for the dynamic analysis of pile groups but in most of them nonlinear behavior of soil is neglected. It was found that during strong ground motions stress concentration occurs in the soil surrounding the piles causing it to behave nonlinearly. Hence for proper dynamic analysis this fact shouldn't be underestimated.

In this paper a new approach to include the nonlinear behavior of soil in pile analysis is presented. For the linear pile analysis a rigorous approach based on boundary integral method or Green's function formulation and proposed by Kaynia & Kausel (1982) is used. While equivalent linearization technique is employed to include the material nonlinearity of the soil media. Different types of soil and both transient and harmonic excitation are considered in the analysis.

## 2. MODELLING

Since stress concentration in soil around the piles remains in a limited zone. Hence as shown in Fig. 1 infinite soil medium is divided into two zones, one is nonlinear zone surrounding the pile group and other is

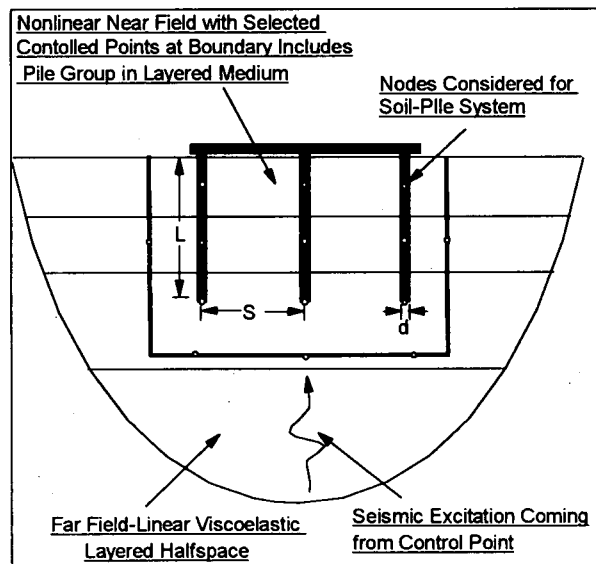


Fig. 1 Model Used for Nonlinear Analysis of Pile Group

remaining infinite soil media. For the purpose, nonlinear zone is divided into a number of layers while linear zone is modeled as a layered halfspace. Property of soil medium may vary from layer to layer. For modeling of nonlinear soil hyperbolic model is used. It is assumed that soil and pile are in perfect contact to each other and no gap or separation is allowed at soil-pile interface.

### 3. FORMULATION

#### (1) Linear Analysis for pile Group

A very rigorous three dimensional approach proposed by Kaynia & Kausel (1982) has been used for the linear analysis. This approach fully takes into account coupling between horizontal and vertical modes of vibration. Effect of pile-soil-pile interaction is also duly considered in this approach which is very important in case of pile group. The soil medium is assumed to be viscoelastic layered halfspace for this part.

In this approach three basic wave equations are solved through Fourier and Hankel transformations and Green's functions. Thus the displacements fields due to uniform barrel and disk loads associated with pile-soil interaction forces are computed. These functions yield the dynamic soil flexibility matrix, which is combined with pile flexibility matrix derived by solving beam equations. In this formulation after doing manipulation and applying boundary conditions one can get a relationship relating forces at pile heads and tips with displacements at these points. This relationship can be described as-

$$P_e = K_e * U_e + Q \quad (1)$$

where  $P_e$  and  $U_e$  are the forces and displacement vector referred to the end of the piles (i.e. pile head and pile tip).  $K_e$  is an equivalent stiffness matrix, which represent dynamic stiffness of soil-pile system and  $Q$  is a load vector due to seismic effects, these are defined as-

$$K_e = [K_p + \psi^T(F_s + F_p)^{-1}\psi] \quad (2)$$

$$Q = -\psi^T(F_s + F_p)^{-1}\psi U_s \quad (3)$$

where  $U_s$  represent seismic displacement in the medium and Matrices

$F_s$  = Soil-flexibility Matrix when there is no pile in the medium

$F_p$  = Pile flexibility Matrix for clamped end pile

$K_p$  = Pile stiffness matrix relating forces at the end of pile with end displacements

$\psi$  = This is a shape function matrix relating displacement at any point of pile with end displacements.

#### (2) Nonlinear Soil Model

Nonlinearity of soil is modeled using hyperbolic model, defined by the following equations:

$$\frac{G}{G_{\max}} = \frac{1}{1+\gamma/\gamma_r} \quad (4)$$

$$\frac{D}{D_{\max}} = \frac{\gamma/\gamma_r}{1+\gamma/\gamma_r} \quad (5)$$

where  $G$  and  $D$  represent the shear modulus and damping at a particular strain  $\gamma$  while  $G_{\max}$  &  $\gamma_r$  represent maximum value of  $G$  & reference strain for the given soil media respectively.

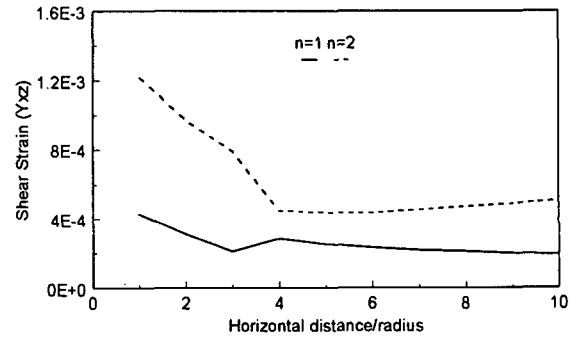


Fig. 2 Variation of Shear Strain with Horizontal Distance from the centre of Pile in top two layers ( $f=10$  Hz).

#### (3) Equivalent Linearization Technique

Assuming some initial soil properties, shear strains are calculated in near field soil. As shown in Fig. 2, it is found that shear strains are decreasing as one go away from the piles. Thus maximum strains lies in near vicinity of the pile, hence strains calculated at a distance 1.5 time the radius of pile will be a good representation of strains in soil media. Further for the case of horizontal vibrations strains are calculated in the direction of loading as well as perpendicular to it and out of these two maximum is taken for the purpose.

For the purpose of seismic analysis first free-field motion is computed at the boundary of the near and far field, with initial soil properties. Now strains are computed in soil media in all layers, taking into account the effect of soil-pile interaction. New property of soil media for each layer is computed for a strain equal to 2/3 of maximum strain. Since the properties of the soil medium is changed hence modified controlled motion is found for new properties. Thus iteration is carried out until the properties of soil medium converged. Nonlinear response will be that one computed using converged properties of soil medium.

### 4. RESULTS

Results presented in this paper are for a typical sand while other data are as follows:

$$E_p/E_s = 500, \rho_s/\rho_p = 0.7, L/d = 15, \nu = 0.4$$

$$G_{\max} = 60 \text{ Mpa}, D_{\max} = 0.29, \gamma_r = 3.6 \times 10^{-4}$$

where  $E_p$  and  $E_s$  represent Young's modulus of pile and initial young's modulus of soil respectively.  $\rho_s$  &  $\rho_p$  represent mass density of soil and pile respectively.  $L$  and  $d$  are length and diameter of pile respectively.  $\nu$  represent Poisson's ratio of soil medium. Initially soil is assumed to be homogeneous.

### (1) Impedance Functions

These are derived by applying lateral harmonic excitation (at pile head or at pile cap in case of group) of a constant force amplitude and noting the complex amplitude of horizontal displacement at the same point. Amplitude of force should be enough that strains computed lies in the range ( $10^{-5}$  -  $10^{-3}$ ). Thus complex impedance function is given by:

$$S(\omega) = P_0/U_0 \quad (6)$$

Indirectly these functions can be computed from equation (1) after removing the seismic load vector Q and applying boundary condition at pile head and at pile tip. Thus a reduced stiffness matrix defining dynamic stiffness at pile head can be obtained. Individual elements of this matrix will represent the impedance function for a particular mode of vibration.

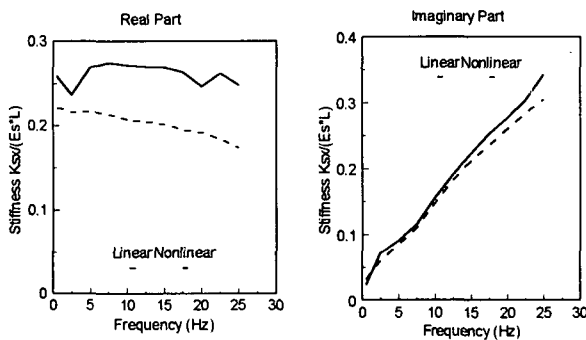


Fig. 3 Linear & Nonlinear Impedance for a Single Pile

Fig. 3 shows the real and imaginary part of impedance function while Fig. 4 also shows same thing but derived for the pile group. In these figures  $K_{sx}$  and  $K_{gx}$  denotes the horizontal impedance function for a single pile and pile group respectively which are normalized using  $E_s$  i.e. young's modulus of soil. Further deriving these results it is assumed that piles are rigidly connected to pile cap. Both figures shows that in general nonlinearity reduces stiffness as well as damping of the soil-pile system. This can be justified by the fact that shear strength of soil is decreased due to the nonlinear effects.

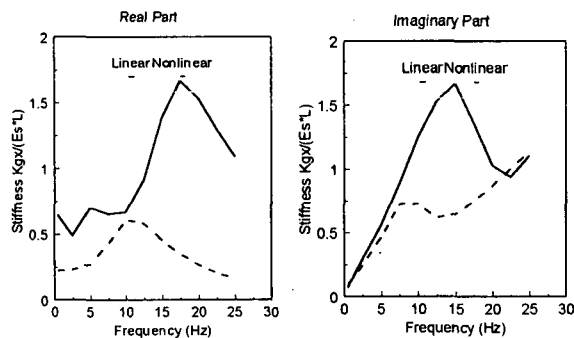


Fig. 4 Linear & Nonlinear Impedance for a 2\*2 Pile Group

### (2) Seismic Response

Seismic excitation is assumed to be consist of vertically propagating shear waves and thus causing horizontal displacements as well as rotation of the pile cap. This controlled motion is assumed to be act at the surface of bedrock or halfspace. Further both harmonic excitation with constant displacement amplitude and real earthquake record, is considered for analysis. In case of harmonic excitation displacement amplitude is assumed to be 0.3 cm which causes the required strains in soil media. For real earthquake, acceleration time history of El-Centro earthquake (N-S component) is used. By applying the correction given by Berg & Housner (1961) displacement time history of earthquake is obtained. Next by the use of FFT, strain time history for each cycle is obtained and selecting maximum strain from this, equivalent linearization is carried out.

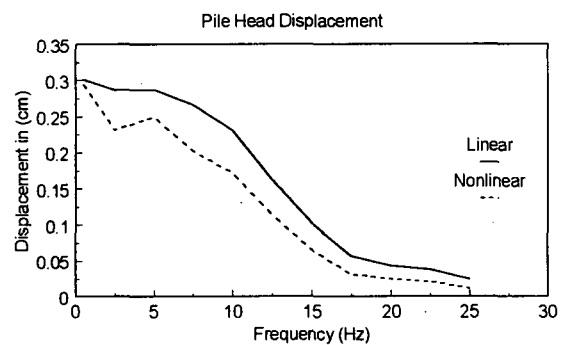


Fig. 5 Linear & Nonlinear Seismic Response of a Single Pile

Fig. 5 shows the effect of nonlinearity on seismic response for a single pile while Fig. 6 shows same but for a pile group. Results obtained are interesting that nonlinearity is decreasing seismic response despite the fact that dynamic stiffness is also decreasing. Similarly Fig. 7 shows the time history of linear and nonlinear seismic response for a real earthquake. In this case nonlinear response is higher compare to linear one. Thus these contrary results need a thorough justification.

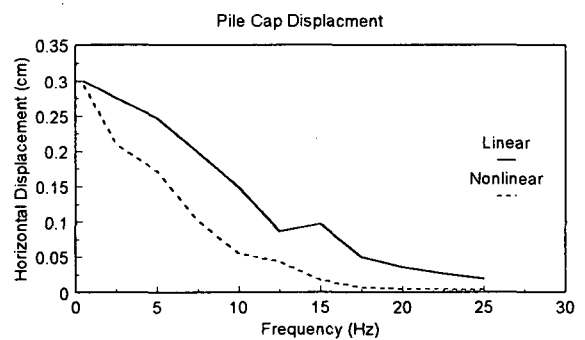


Fig. 6 Linear & Nonlinear Seismic Response of a 2\*2 Pile Group

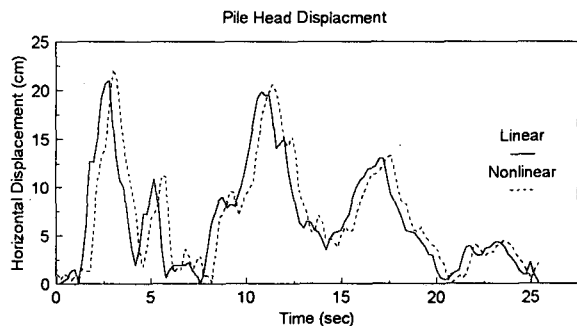


Fig. 7 Linear & Nonlinear Seismic Response of a Single Pile Due to El-Centro Earthquake (NS component)

### (3) Justification of Results

In fact computation of seismic response incorporates following three things which effects the response:

#### a) Free-field motion

Various one dimensional wave theories for example Wolf (1985) suggests that amplification of motion is higher for a stiffer soil medium compare to softer one. Thus will be higher for linear case compare to nonlinear one.

b) **Dynamic Stiffness**- as defined by equation 2 , which is higher for stiffer medium.

c) **Seismic load vector Q**- defined by Eq. 3 which itself depends on free-field motion and dynamic stiffness.

Again it is noted that all the above quantities are frequency dependent hence seismic response too. If we check the variation of horizontal transfer function with frequency for a typical soil assuming linear conditions, Kaynia & Kausel (1982), then it is found that for very low frequency it remains almost unity and then increases in low range of frequency and at higher frequency decreases rapidly. Again the fact that in the considered range of frequency for a stiffer medium transfer function is almost unity while for a softer medium it is considerably lower than unity, also justifies that nonlinear seismic response may be less than linear one. In other words it can be said that effect of pile-soil interaction or in general soil-structure interaction is higher for weaker soils and thus decreasing transfer functions for nonlinear case.

Further result obtained for real earthquake is also justified by the fact that frequency range considered for this is comparatively lower. Here soil medium selected for analysis is a typical sand with ( $\nu = 0.4$  &  $G_{max} = 60MPa$ ) . Analysis is carried out for clay medium also, for that case also same trends are observed.

## 5. CONCLUSIONS

1. Material nonlinearity of the soil medium decreases the dynamic stiffness of soil-pile system. Decrease in stiffness is higher compare to damping.

2. Effect of nonlinearity on seismic response is very much dependent on frequency and other factors. For considerable range of frequency nonlinear seismic response is smaller than linear one.

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