Real-Time Tsunami Simulations with Surrogate Modeling

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There have been many numerical models proposed for tsunami propagation simulations. They are all very numerically intensive, requiring days to complete the calculations. For probabilistic risk analysis, which requires many simulations, these methods are infeasible. Therefore, in this paper, we propose a framework of surrogate modeling, which introduces the potential of real-time risk assessment. To create the surrogate model, we calculated the prominent modes of the given data, after which we used RBF interpolation for mapping the input data points to their corresponding coefficients. The surrogate model this constructed is expected to be accurate, given the simulation cases used are sufficiently dense for the complexity of the results.

1 . Introduction

Because tsunamis are rare events, it is often difficult or even impossible to collect sufficient data for tsunami forecasts. The small number of observations is one of the reasons for uncertainty in describing earthquake sources. That is where computer modeling can provide more in-depth insight into tsunami generation and risks. Tsunami simulations have been used to study past events as well as predict the effects of future events. In recent years, numerical models for tsunami simulations have become more complex and accurate. One of the drawbacks of these methods is often computation time. Surrogate modeling can be used to reduce the computational time of these tasks by replacing expensive numerical simulations with approximate functions that are much faster to evaluate. When a model must be assessed quickly or when many repeated model evaluations are required, surrogate modeling is effective. The primary objective of this study is to develop a method to realize real-time tsunami simulations by using a surrogate model when an earthquake occurs in the study area. A POD-RBF method is applied to the simulation results performed by Koshimura et al. with various earthquake scenarios in the Nankai Trough.

2 . The earthquake source model

The Okada model is a commonly used earthquake source model, which assumes an elastic half-space containing the fault. The initial sea-floor deformation is computed using this model, while the tsunami propagation uses the depth-averaged shallow water equations. The Okada model has nine input parameters, as shown in Figure 1. The following equation defines the magnitude:

$$M_w = \frac{2}{3}(log_{10}M_0 - 9.1) \;, \tag{1}$$

where M_0 is the fault area multiplied by displacement and material rigidity. Two magnitudes were used for our POD-RBF model.

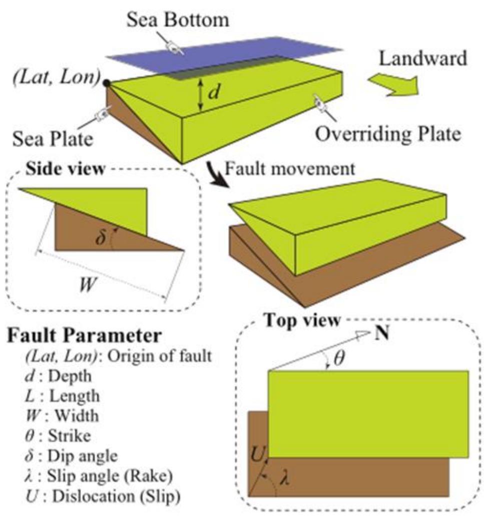

Figure 1. Input parameters for the Okada model

3. The depth-averaged shallow water equations

For the depth-averaged shallow water equations, we simplify the Navier-Stokes equations to use depth-averaged velocity. The general form of the shallow water equations can be expressed as:

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{A}_i \frac{\partial \mathbf{u}}{\partial x_i} - \frac{\partial}{\partial x_i}\left(\mathbf{K}_{ij}\frac{\partial \mathbf{u}}{\partial x_j}\right) - \mathbf{r} = \mathbf{0} \;, \tag{2}$$

$$\mathbf{A}_1 = \begin{bmatrix} 0 & 1 & 0 \\ -u_1^2 + gh & 2u_1 & 0 \\ -u_1u_2 & u_2 & u_1 \end{bmatrix}, \quad \mathbf{A}_2 = \begin{bmatrix} 0 & 0 & 1 \\ -u_1u_2 & u_2 & u_1 \\ -u_2^2 + gh & 0 & 2u_2 \end{bmatrix}, \tag{3}$$

$$\mathbf{K}_{11} = \begin{bmatrix} 0 & 0 & 0 \\ -2\nu u_1 & 2\nu & 0 \\ -\nu u_2 & 0 & \nu \end{bmatrix}, \quad \mathbf{K}_{12} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -\nu u_1 & \nu & 0 \end{bmatrix}, \tag{4}$$

$$\mathbf{K}_{21}=\begin{bmatrix}0 & 0 & 0\\ -\nu u_2 & 0 & \nu\\ 0 & 0 & 0\end{bmatrix},\quad \mathbf{K}_{22}=\begin{bmatrix}0 & 0 & 0\\ -\nu u_1 & \nu & 0\\ -2\nu u_2 & 0 & 2\nu\end{bmatrix}, \quad (5)$$

$$\mathbf{u}=\begin{bmatrix}h\\ u_1 h\\ u_2 h\end{bmatrix},\quad \mathbf{r}=\begin{bmatrix}0\\ -gh\dfrac{\partial z}{\partial x_1}-f_b u_1 h+f_c u_2 h\\ -gh\dfrac{\partial z}{\partial x_2}-f_b u_2 h-f_c u_1 h\end{bmatrix}, \quad (6)$$

where $\mathbf{u}$ is the vector of conservation variables, n is Manning's roughness coefficient, g is gravitational acceleration, ν is horizontal kinematic viscosity, z is the height of the bottom surface from the mean water level, and H is the total water depth. Matrices $\mathbf{A}_i$ and $\mathbf{K}_{ij}$ are derived from the Euler and viscous flux vectors, and $\mathbf{r}$ contains other components such as external forces.

4. Proper orthogonal decomposition – radial basis function (POD-RBF) surrogate modeling

Proper orthogonal decomposition, hereafter POD, is a dimension reduction technique, which minimizes the Frobenius norm of the difference between the original dataset and the reduced-order model. Radial basis function interpolation is a set of techniques used for mapping a set of outputs and inputs. This set of interpolation techniques is defined by functions based on the distance between the input data and points defined at training.

Given a data matrix $\mathbf{M}$ with the observations from each of the N cases in data vectors $\mathbf{M}_i$ (i = 1, …, N). We can decompose it into modes as follows.

$$\mathbf{M}=\mathbf{W}\boldsymbol{\Sigma}\mathbf{V}^T \quad (7)$$

The diagonal entries of $\boldsymbol{\Sigma}$ are the singular values of $\mathbf{M}$, $\mathbf{W}$ is the matrix of left-singular vectors and $\mathbf{V}$ is the matrix of right-singular vectors. We can relate this decomposition to eigenvalue decomposition, which can only be applied to diagonalizable matrices. The following relations hold for the above equation:

$$\mathbf{M}^T\mathbf{M}=\mathbf{V}\boldsymbol{\Sigma}^T\mathbf{W}^T\mathbf{W}\boldsymbol{\Sigma}\mathbf{V}^T=\mathbf{V}(\boldsymbol{\Sigma}^T\boldsymbol{\Sigma})\mathbf{V}^T \quad (8)$$

$$\mathbf{M}\mathbf{M}^T=\mathbf{W}\boldsymbol{\Sigma}^T\mathbf{V}^T\mathbf{V}\boldsymbol{\Sigma}\mathbf{W}^T=\mathbf{W}(\boldsymbol{\Sigma}^T\boldsymbol{\Sigma})\mathbf{W}^T \quad (9)$$

The right sides of the above equations correspond to the eigenvalue decompositions of $\mathbf{M}^T\mathbf{M}$ and $\mathbf{M}\mathbf{M}^T$. In our research, we use Low-rank matrix approximation, where only a subset of the modes obtained is used for the surrogate model.

$$\tilde{\mathbf{M}}=\mathbf{W}\tilde{\boldsymbol{\Sigma}}\mathbf{V}^T \quad (10)$$

Here, $\tilde{\boldsymbol{\Sigma}}$ contains the r largest singular values obtained earlier. If we express this as a summation, we obtain the following equation;

$$\tilde{\mathbf{m}}_i=\sum_{k=1}^{r}\alpha_{ik}\mathbf{w}_k=\alpha_{i1}\mathbf{w}_1+\cdots+\alpha_{ir}\mathbf{w}_r \quad . \quad (11)$$

We can further estimate the simulation results of other inputs by rewriting the equation and creating a function that seamlessly maps the inputs to the coefficients for each of our modes.

$$\tilde{\mathbf{m}}(\mathbf{x})=\sum_{k=1}^{r}f_k(\mathbf{x})\mathbf{w}_k \quad (12)$$

For this function $f(\mathbf{x})$ that maps the values of the inputs to the coefficients α_{ik} in the above equation, we use a radial basis function which is defined as follows,

$$f(\mathbf{x})=\sum_{i=1}^{N}c_i\exp(-\gamma||\mathbf{x}-\mathbf{x}_i||^2) \quad . \quad (13)$$

We use least squares regression to obtain the values of so the c_i function fits the input-output pairs.

5. Simulation data

To train our POD-RBF model, we used simulation results compiled by Koshimura et al. For these simulations, various different parameters obtained by Igarashi et al[2]. (2016) were combined to create 666 scenarios. The parameters used are shown in Table 1, and the locations of the faults are shown in Figure 2.

Table 1. Fault parameters

Mw	8.2, 8.5
Strike (degree)	200, 240, 315
Dip (degree)	5, 15, 20, 25
Slip (degree)	90, 150
L (km)	177.8, 251.2
W (km)	89.1, 125.9
Depth (km)	5, 20
Dislocation (m)	10.0, 14.1

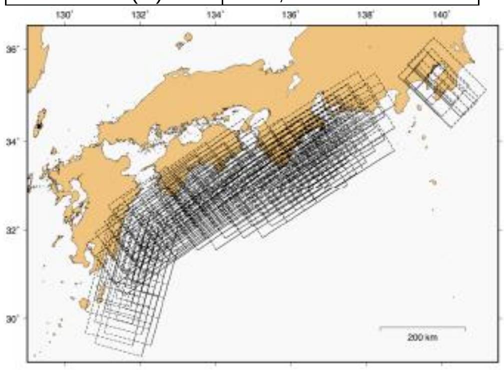

Figure 2. Distribution of fault planes

These parameters were used as inputs for the POD-RBF model. We deemed the magnitude moment and strike angle most important for constructing the surrogate model as well as the locations of the active faults.

6. Concluding remark

We have developed a framework of surrogate modeling for real-time tsunami simulations by making use of the POD-RBF method.

References

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