# ESTIMATION OF HORIZONTAL VIBRATION FREQUENCY OF LIQUEFIED SAND DEPOSIT

Institute of Industrial Science, The University of Tokyo Member ○ Mladen V. KOSTADINOV
The University of Tokyo Member Ikuo TOWHATA
Institute of Industrial Science, The University of Tokyo Member Ken SUDO

## Introduction

Seismic records from liquefied-soil sites reveal that the frequency of the horizontal ground acceleration suddenly drops down below 1.0 Hz after initiation of the phenomenon, while no change is observed in the vertical acceleration. This alteration is the main feature of the ground motion at liquefied sites and is utilized in several methods for liquefaction judgment from seismic records (e.g. Kostadinov and Yamazaki, 2001). Present paper discusses an attempt to estimate the vibration frequency of a liquefied sand deposit using one-dimensional ground response analysis.

## Vibration frequency of a sand deposit at onset of liquefaction

Horizontal seismic motion of a sand deposit is approximated up to the initiation of liquefaction through the linear wave-propagation approach, together with two key assumptions: (1) at the onset of liquefaction the sand deposit is in a state of resonance with the seismic excitation and vibrates in its fundamental (first) mode; and (2) liquefaction occurs when the maximum shear strain in the soil exceeds a critical value, $g_{\text{liq}}$. This approach was effectively applied by Towhata et al. (1996) for estimation of the minimum spectrum intensity, associated with the soil liquefaction.

Consider a single uniform layer of homogenous, isotropic, linear elastic cohesionless soil, subjected to a one-dimensional upward harmonic shear wave (Fig. 1). The lateral motion of the soil particles $u(z,t)$ can be expressed as

$$u(z,t) = D_{\max} \cos\left(\frac{wz}{v_s}\right) \exp\{iwt\} \qquad (1)$$

where $D_{\max}$ is the maximum displacement amplitude of the soil particles, $w$ is the circular frequency of the excitation at the layer base, $v_s$ is the shear-wave velocity and $z$ stands for the depth.

Using the first assumption, the frequency of vibration of the soil layer at resonance $f_{\text{liq}}$ is expressed through the natural frequency of the layer fundamental mode and further transformed as

$$f_{\text{liq}} = \frac{v_s}{4H} = \sqrt{\frac{A_{\max}}{8pg_{\text{liq}}H}} \qquad (2)$$

where $H$ is the depth of the layer and $A_{\max}$ is the maximum surface acceleration. Considering $A_{\max}$ = 1.0 m/s$^2$, $H$ = 2 ~ 15 m, $g_{\text{liq}}$ = 0.01875 in Eq. (2), the vibration frequency of the sand layer $f_{\text{liq}}$ is estimated to be 0.4 ~ 1.0 Hz, where the lower value corresponds to the larger depth.

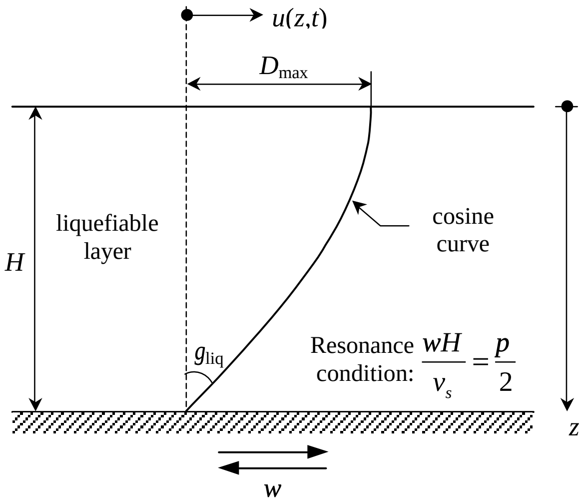


**Fig. 1.** Approximation of the response of a single-layered sand deposit, subjected to upward shear wave.

Above result can be enhanced by considering an unliquefiable dry soil layer above the liquefiable one. If the surface layer is assumed to be much stiffer than the underlying layer, it can appear as a rigid mass above the latter. Let denote with $r_m = r_s h / r_t H^*$ the ratio of the surface-layer mass to the liquefiable-layer mass, where $h$ and $r_s$ are the depth and the mass density of the surface layer, while $H^*$ and $r_t$ are the depth and mass density of the subsurface liquefiable layer. Combining the new resonance condition with the expression for $r_m$ and solving approximately the resultant transcendental equation, the vibration frequency of the sand deposit, when a surface layer is considered, $f_{\text{liq}}^*$, is

$$f_{\text{liq}}^* = k_f \, f_{\text{liq}} \qquad (3)$$

where $k_f = k_f(r_m)$ is a frequency reduction coefficient.

Variation of $k_f$ with respect to the mass layer ratio is shown in Fig. 2. In the case $r_m$ is equal to unity, vibrating frequency of the sand deposit will drop as low as 0.55(0.4 ~1.0) = 0.2 ~ 0.7 Hz. In practice, the layer mass ratio is less than unity.

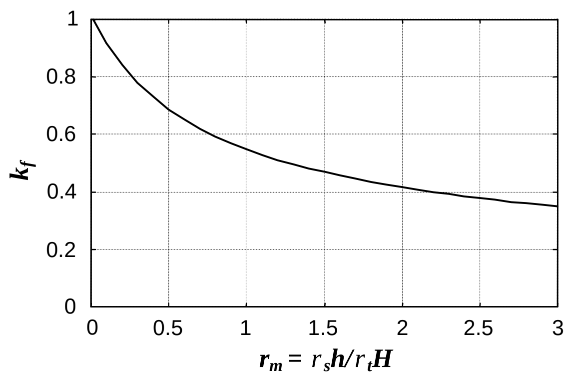


**Fig. 2.** Relationship between frequency reduction coefficient $k_f$ and layer mass ratio $r_m$.

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***Contact Address*:** 4-6-1 Komaba, Meguro-ku, Tokyo 153-8505, Tel. 03-5452-6098 Ext. 58064, Fax 03-5452-6386

**Table 1.** Comparison between calculated vibration frequency $f_{liq}$ and *MIF* for Kawagishi-cho and Port Island seismic records.

| Station | Component | $A_{max}$ (cm/s$^2$) | $f_{liq}$ (Hz) | $MIF(t_{PGA})$ (Hz) | $H$ (m) |
|---|---|---|---|---|---|
| Kawagishi-cho | EW | 170.9 | 0.4 | 1.2 | 20.00 |
| | NS | 128.9 | 0.4 | 1.0 | |
| Port Island | EW | 284.3 | 0.6 | 1.3 | 19.00 |
| | NS | 341.2 | 0.6 | 0.8 | |

$t_{PGA}$ – time instant at which peak ground acceleration takes place.

**Comparison with earthquake case histories**

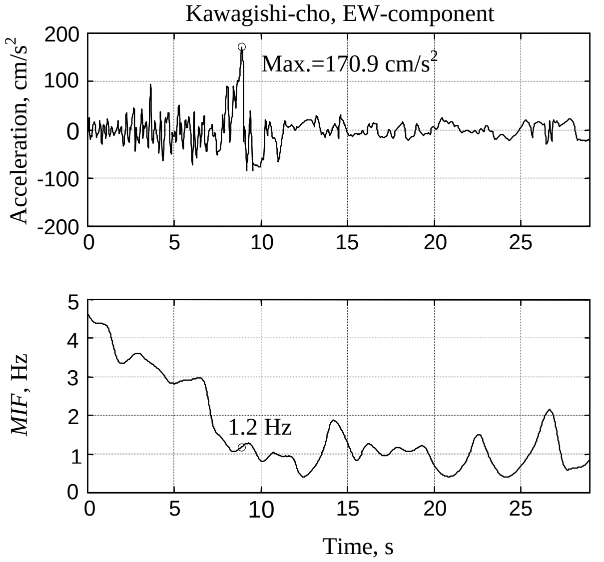
Equation (2) is examined against ground motion records, obtained at sites, where liquefaction has occurred. These are Kawagishi-cho, 1964 Niigata earthquake, and Port Island, 1995 Hyogo-ken Nanbu earthquake, both dramatic examples of liquefaction-induced damages.

Figure 3(a) depicts the EW-component of the Kawagishi-cho record. The peak ground acceleration (*PGA*) occurs just before the elongation of the shaking period that supports the assumption for resonance. Acceleration time history is processed with the short-time Fourier transform (STFT) and then squared. From the resultant spectrogram is computed the mean instantaneous frequency, *MIF*, which shows the evolution of the ground shaking period with the time. Clearly is seen the frequency drop that has started around the seventh second. The value of *MIF* that corresponds to occurrence of *PGA* is 1.2 Hz.

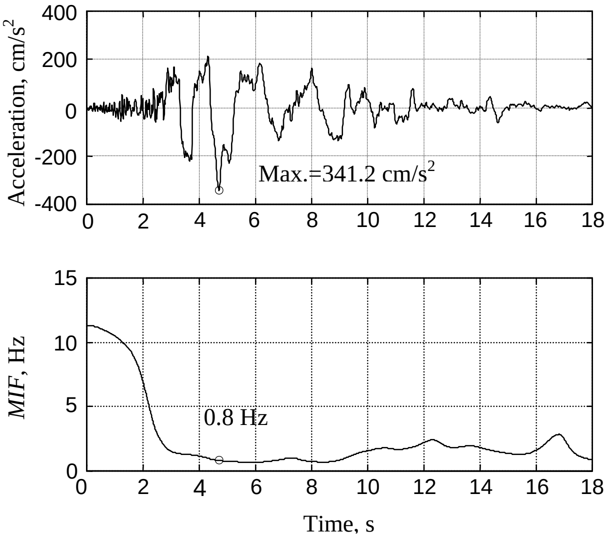
The depth of the liquefiable layer $H$ is determined from the soil profile of the site as a sum of consequent sand sub-layers, starting from the surface. By substituting it together with $A_{max}$ in Eq. (2), $f_{liq}$ is estimated as 0.4 Hz. The spike-like acceleration peak and a shallower liquefaction depth could explain this difference.

Figure 3(b) depicts the NS-component of the Port Island record. Here, *PGA* takes place when the long-period motion has already started and therefore *MIF* at the peak acceleration is low - 0.8 Hz. The corresponding value of $f_{liq}$ is 0.6 Hz.

Results for all components of the discussed records are given in Table 1.




(a)




(b)

**Fig. 3.** Time histories of ground acceleration and corresponding mean instantaneous frequency, *MIF*: (a) Kawagishi-cho, EW component and (b) Port Island, NS component.

**Discussion**

Estimated vibration frequency of a liquefied sand deposit using Eq. (2) provides a reasonable approximation of the observed one from the seismic records. In general, Eq. (2) gives a lower value, since the real depth of the liquefied deposit is not clear. Assumption for the resonance is not always proved and therefore the peak ground acceleration is often caused by a spike-like motion.

Despite this, the threshold of the horizontal shaking frequency that implies soil liquefaction can be assessed satisfactory using Eq. (2), the minimum *PGA* associated with the phenomenon and reasonable values of the liquefied-soil depth. Obtained ranges for $f_{liq}$ are in agreement with the limit values for the frequency ground motion parameters, employed in the liquefaction detection method.

**References:**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