

Study on Constitutive Model Considering Hardening Process and Loading History for Early Age Concrete



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1. Introduction

Cracking of early age concrete, such as thermal cracking in massive concrete and cracking resulting from drying shrinkage or autogenous shrinkage, not only forms mechanical weaknesses but also causes a reduction in durability. It is therefore important to quantitatively evaluate the cracking properties of early age concrete. In order to predict the cracking properties of a concrete structure by numerical analysis, it is urgently necessary to develop a constitutive model that is capable of simultaneous quantitative treatment of both (1) time dependency in the mechanical properties of concrete including softening after cracking and (2) differences in the external loading histories acting on concrete.

2. Outline of Proposed Model

2.1 Stress-strain curves, loading history and volume function

The developed cracking model is based on the solidification theory proposed by Bazant et al. (1989). Global response of concrete depends on each solidified element that represents the physical phenomenon at the micro-scale. The material properties of concrete in the hydration process depend on the volume function that sequentially increases. The stress-strain relationship at a certain point of time is given as the sum of stress-strain relationships of solidified elements at each time step. The global stress at time t_3 shown in Figure 1, for instance, is calculated as follows:

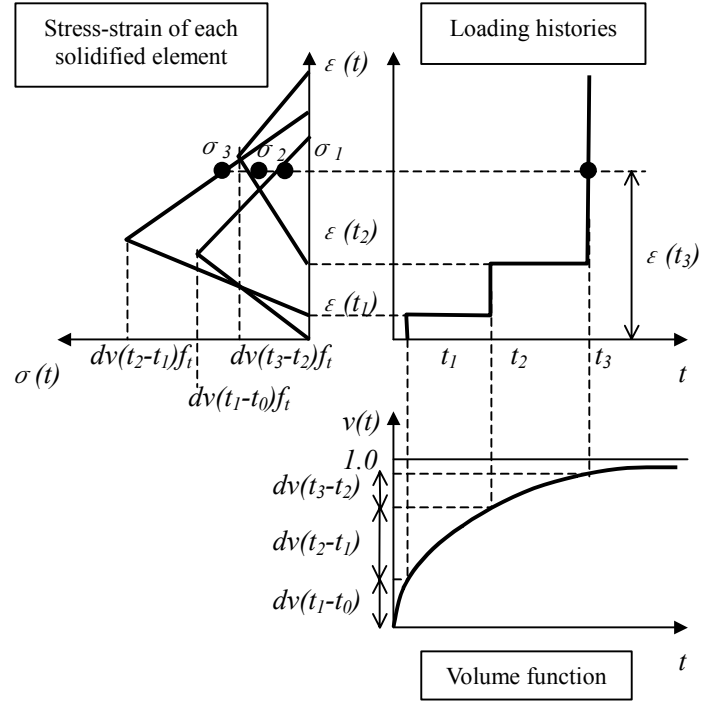


Figure 1 Basic concept on proposed constitutive model.

$$\sigma_g(t_3) = dv(t_1 - t_0)\sigma(\varepsilon(t_3)) + dv(t_2 - t_1)\sigma(\varepsilon(t_3) - \varepsilon(t_1)) + dv(t_3 - t_2)\sigma(\varepsilon(t_3) - \varepsilon(t_2)) \quad (1)$$

where $\sigma_g(t_3)$ is the total stress at time t_3 ; t_i ($i=0-2$) is the time changing in loading histories; $\varepsilon(t_i)$ is strain at t_i ; and $dv(t_{i+1}-t_i)$ is the increment in the volume function.

2.2 Definition of volume function

Figure 2 and Eq. (2) show the volume function for time dependency in the physical property values formulated and normalized with respect to the 28day values.

$$v(t) = \frac{t}{(t + 0.6) \times 0.979} \quad (2)$$

Where t is the total time (days). This shows that the development of physical properties, such as the elastic modulus, flexural strength, and fracture energy, are all expressible by this equation.

2.3 Method of determining tension softening curves and their superposing

A method was adopted for uniquely determining the bi-linear tension softening curves depending on the volume function, which is based on 1/4 model. Both tensile strength and stress at breaking point depended on the obtained volume function. However, the crack width at breaking point and critical crack width were constant values of 0.12 and 0.012, respectively. As stated above, this method is characterized by the stress changing over time and the crack width being constant independently of age. It was confirmed that, if the volume function can be specified, then the tensile

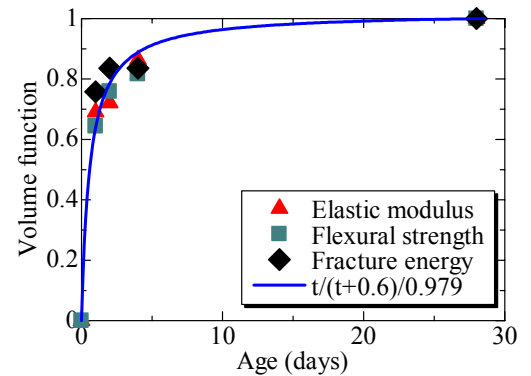
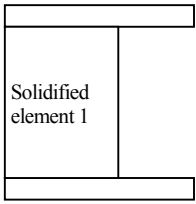
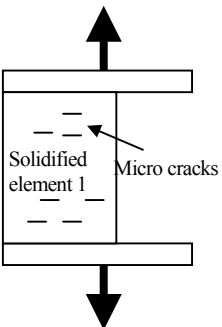
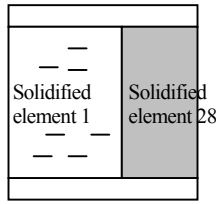
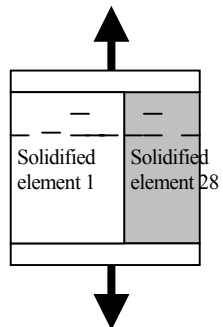


Figure 2 Estimated volume function in the present experiment.

Loading histories	(a) 1 day	(b) Loading and un-loading at 1 day (1st loading)	(c) Re-curing (up to 28 days)	(d) Loading at 28 days (2nd loading)
Solidified elements*	Solidified element 1: generated Solidified element 28: none	Solidified element 1: damaged Solidified element 28: none	Solidified element 1: damaged Solidified element 28: generated	Solidified element 1: damaged Solidified element 28: damaged
Concrete element				

* Solidified element 1: element generated by 1 day
Solidified element 28: element generated by 28 days

Figure 3 Images of Solidified Elements and Loading Histories

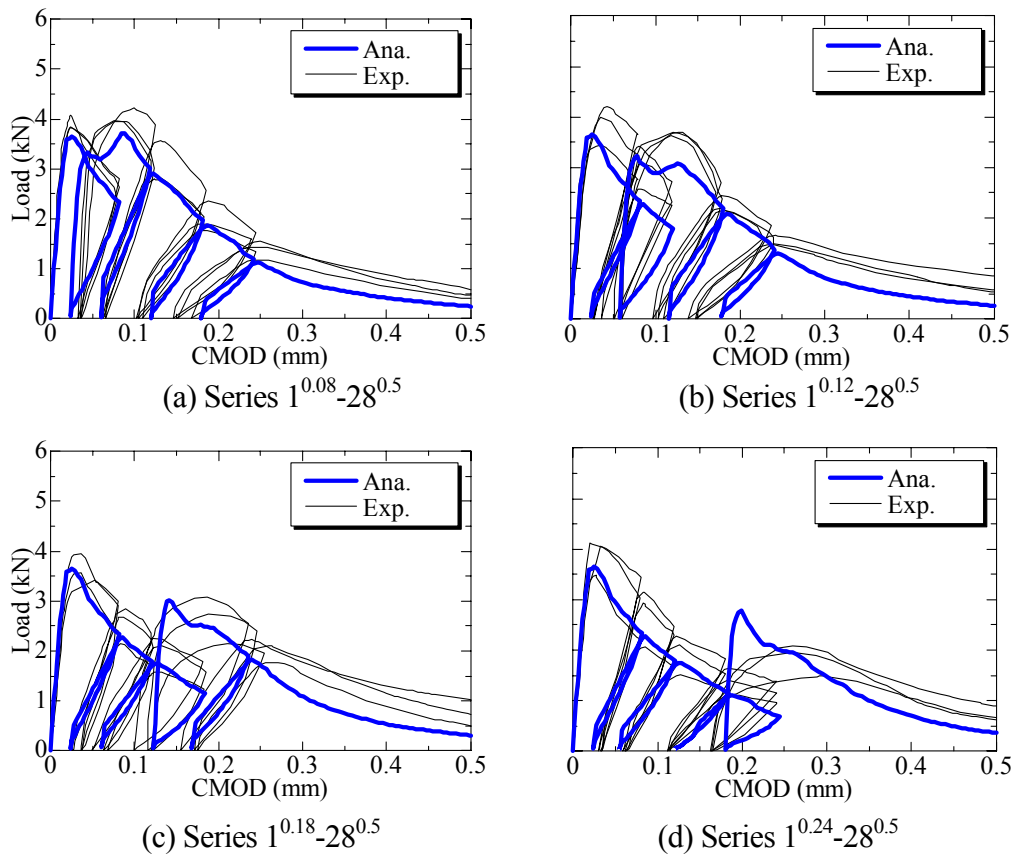


Figure 4 Analytical Load-CMOD Curves (first loading: 1day, second loading: 28days).

constitutive model at a given time can be specified.

Regarding the superposing the stress-strain curves, the curves at 28days, which represents global response of concrete element, can be calculated by using the residual stress-strain curve of solidified element 1 with damage (1day) and stress-strain curve of solidified element 28 without damage (28days) generated during the re-curing, as shown in Figure 3.

3. Analytical Results

Figures 4 and 5 show the analysis and test results of specimens subjected to the first loading at 1 and

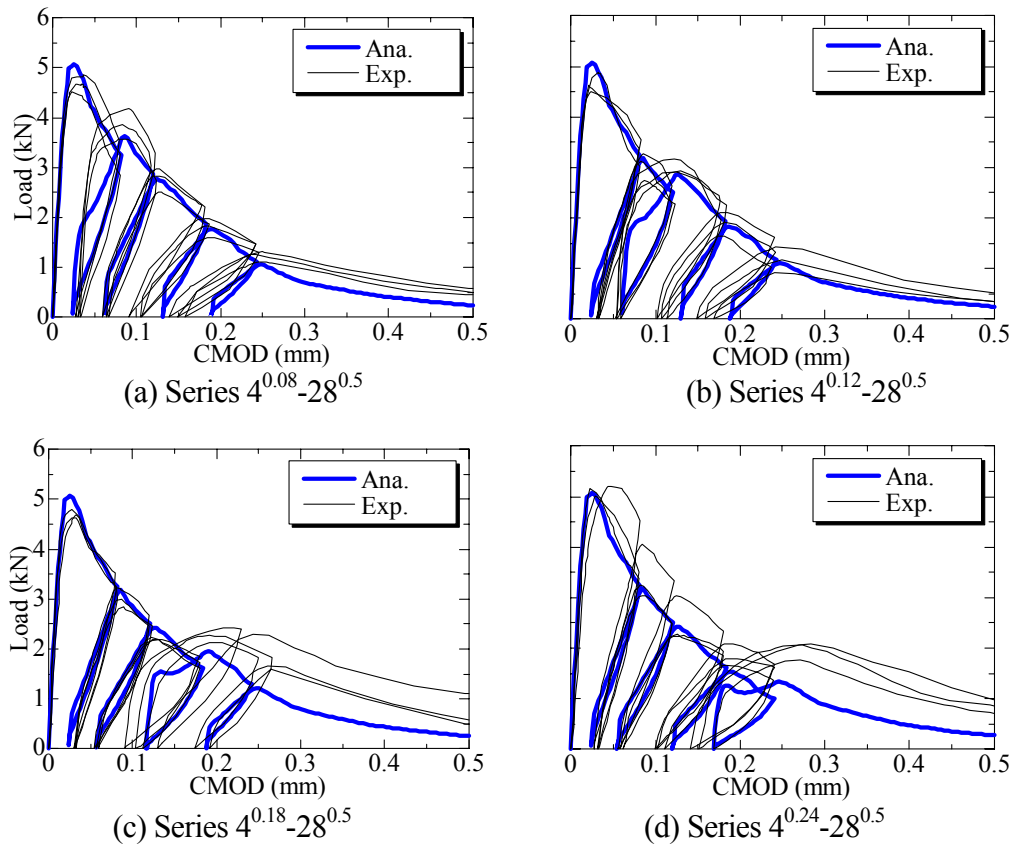


Figure 5 Analytical Load-CMOD Curves (first loading: 4days, second loading: 28days).

4days, respectively, and second loading at 28days. Here, Series $1^{0.08}-28^{0.5}$ means that the first loading at 1day and second loading at 28days were conducted up to CMOD of 0.08mm and 0.5mm, respectively. The shapes of the overall load-CMOD curves are roughly well-estimated, including the behavior in which the peaks under the second loading are increased by re-curing after the first loading.

According to the analysis, the total energy consumed by the time of fracture at 28 days is nearly constant with or without the damage and independently of the damage history. As shown in Figure 5 (d), the behavior under the first loading is basically well-estimated, but the load and energy absorption under the second loading are reproduced as being slightly greater than the test values. This is presumably because the damage to a high degree under the first loading (approximately 0.24 mm in this study) adversely affects the subsequent hardening process of specimens.

4. Concluding Remarks

In this study, flexural loading tests on specimens with different loading histories were conducted to verify the validity of a constitutive model capable of incorporating the hardening process and loading histories of concrete. The authors intend to pursue a method of specifying a more precise volume function, as well as to investigate the unloading path of the model, the effect of creep, and the applicability to other types of concretes, to develop a more generalized constitutive model.

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