

# New Method of Calculation of Long-term Deflection in the 2012 Standard Specification for Concrete [Design]

## 1 Background to the Revised Method of Calculation

The conventional method of calculating the long-term deflection was to model a structural member as a single linear member, and to calculate the long-term displacement and deformation based on the creep strain using a linear creep law. However, E. Witchkreangkrai et al. reported that the deflection of long span PC bridges exceeds the design values and continues to increase<sup>1)</sup>. It has become clear that it is difficult to accurately evaluate the long-term deflection of full-scale structures using the conventional shrinkage strain prediction formulae based on small-scale laboratory creep tests.

## 2 Basic Concept and New Method of Calculation

In the 2012 Specifications, the fundamental principle for the calculation of long-term displacement and deformation produced by the interaction of various physico-chemical phenomena that progress in time and space is to use a method that faithfully takes into consideration the relevant microscopic phenomena as much as possible. Such methods include detailed calculation methods (for example, multi-scale integrated analysis), or simplified calculation methods using new shrinkage strain prediction equations based on multi-scale integrated analysis.

## 3 Calculation of Long-term Displacement and Deformation

### (1) Detailed calculation method: Multi-scale integrated analysis

In multi-scale integrated analysis, the environmental state quantities of temperature and humidity are calculated as solutions satisfying equations of conservation of mass, energy, and momentum corresponding to each time point and each location. This calculated information is passed to a rheology model in accordance with the moisture behavior properties at each void scale. Then the acting stress information together with the creep deformation at each location is calculated. In other words, in multi-scale integrated analysis, autogenous shrinkage, drying shrinkage, and creep itself determine the structural response calculated as the behavior of a substance having microstructure.

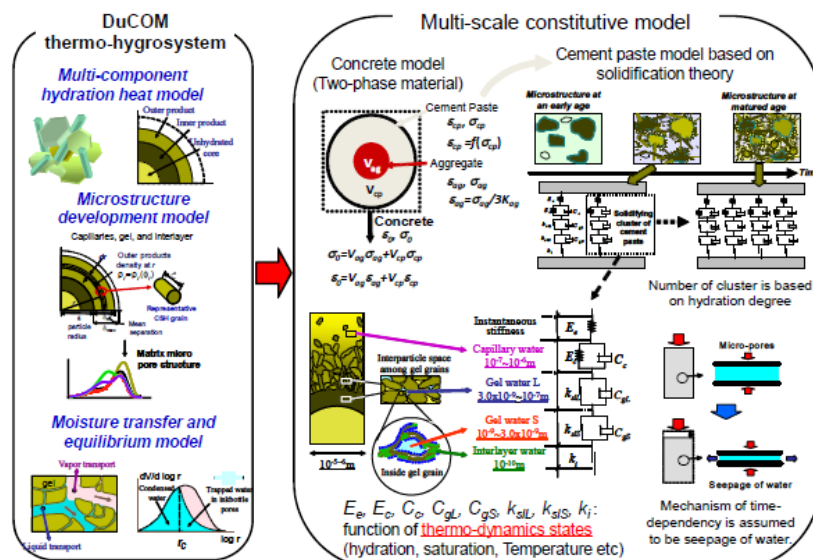


Fig. 1 Scheme of time-dependent modeling associated with thermo-hydro states<sup>2)</sup>

M. Ohno reported that it is possible to accurately calculate the long-term deflection of PC long span bridges, as shown in Fig. 2, using multi-scale integrated analysis that can take into consideration the mutual coupling of phenomena at different dimensions ranging from the thermodynamic state within microscopic voids up to the macroscopic structural response<sup>3)</sup>.

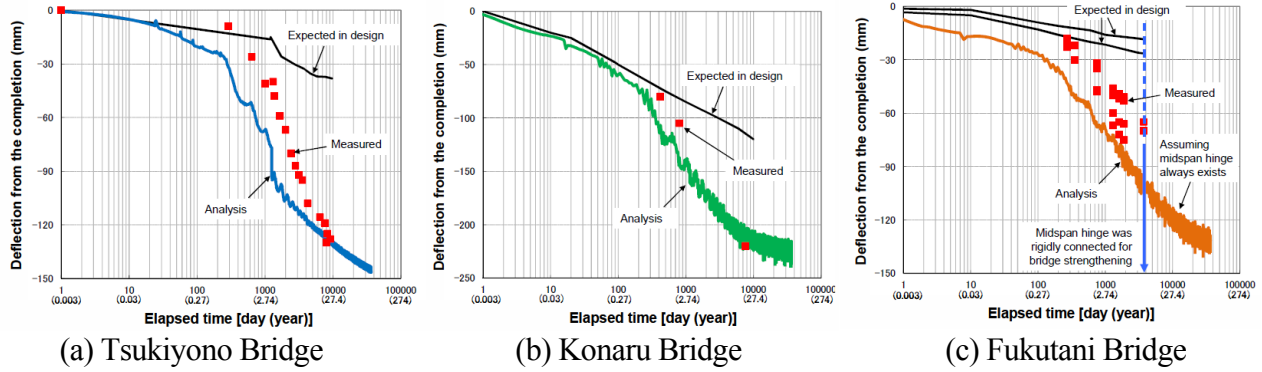


Fig. 2 Scheme of time-dependent modeling associated with thermo-hydro states<sup>3)</sup>

(2) Simplified calculation methods: Structural analysis methods using fiber models

Calculation methods for simply obtaining the long-term displacement and deformation divide a member cross-section into several parts taking into consideration mechanical resistance and environmental conditions (temperature, humidity). The effect of concrete shrinkage and creep are applied as input values in accordance with the environmental conditions for each part. Then the displacement and deformation is calculated from the curvature produced by the difference in shrinkage within the cross-section using the following calculation methods (i) to (iii).

(i) Method of calculating the long-term deflection of a concrete member

The long-term deflection of concrete is calculated taking into consideration the effect of concrete shrinkage and the effect of creep. Normally it may be calculated from equation (1).

$$\delta_t = \delta_L \cdot \phi_t + \delta_{SH} \quad (1)$$

Where  $\delta_t$ : long-term displacement,  $\delta_L$ : displacement due to external forces,  $\phi_t$ : creep coefficient used for long-term deformation calculation, and  $\delta_{SH}$ : displacement due to shrinkage.

(ii) Method of calculating the displacement due to concrete shrinkage

As shown in Equation (2), in accordance with the member age, the displacement due to concrete shrinkage is calculated from the second order integral of the curvature produced by the difference in shrinkage strain arising from the difference in drying conditions and the constraint conditions of the steel members in each part of the cross-section.

$$\delta_{SH} = \iint \phi_{SH} dx dx \quad (2)$$

Where,  $\delta_{SH}$ : displacement due to shrinkage strain,  $\phi_{SH}$ : curvature due to shrinkage strain

Also, the change of the concrete shrinkage strain is obtained by superimposing coefficients representing the change of the concrete drying shrinkage strain and the change of the shrinkage under sealed conditions. In the case of normal concrete using ordinary cement, this may be calculated from Equation (3).

$$\varepsilon'_s(t) = \varepsilon'_{ds}(t, t_0) + \varepsilon'_{as}(t, t_s) \geq 0 \quad (3)$$

Where  $\varepsilon'_s(t)$  : concrete shrinkage strain ( $\times 10^{-6}$ ) at member age  $t$ ,  $\varepsilon'_{ds}(t, t_0)$  : strain ( $\times 10^{-6}$ ) due to concrete drying shrinkage at member age  $t$ ,  $\varepsilon'_{as}(t, t_s)$  : concrete shrinkage strain ( $\times 10^{-6}$ ) under sealed conditions from start of setting until the member age  $t$ . Note that the strain due to concrete drying shrinkage at member age  $t$ , and the shrinkage strain under concrete sealed conditions up to member age  $t$  may be calculated using Equations (4) and (5) respectively.

a) Strain due to concrete drying shrinkage at member age  $t$

$$\varepsilon'_{ds}(t, t_0) = \kappa_{ds} \cdot \varepsilon'_{dsh\infty} [1 - \exp\{-\alpha_{ds}(t - t_0)^{\beta_{ds}}\}] \quad (4)$$

$$\varepsilon'_{dsh\infty} = \frac{980}{1 + 0.53e^{-0.23(W/C-50)}} \times \left( 1 + \frac{RH - 65}{15} \cdot (-0.25 - 7400e^{-0.214W/C}) \right)$$

$$\alpha_{ds} = \frac{1}{(V/S)^2} \left( 200 - 2.7 \frac{W}{C} \right) \quad \text{where } \alpha_{ds} \geq 0.0002$$

$$\beta_{ds} = 0.0074 W/C + 0.00058 V/S + 0.25$$

$$\kappa_{ds} = 1 + \alpha_{ds}(V/S - 25)$$

$$\alpha_{ds} = 0.00018 W/C - 0.0085 \quad (35 \leq W/C \leq 45\%), \alpha_{ds} = -0.00038 \quad (45 \leq W/C \leq 60\%)$$

Where  $\varepsilon'_{dsh\infty}$  : final value of the shrinkage strain ( $\times 10^{-6}$ ) due to drying shrinkage of a  $100 \times 100 \times 400$  mm square prism,  $t$  : concrete member age (days),  $t_0$  : member age at start of drying shrinkage (days),  $\alpha_{ds}$ ,  $\beta_{ds}$  : coefficient representing the drying shrinkage strain progression properties,  $\kappa_{ds}$  : coefficient related to the final value of the drying shrinkage strain,  $W/C$  : water cement ratio (%),  $V/S$  : volume to surface area ratio (mm),  $RH$  : relative humidity (%).

b) Shrinkage strain under concrete sealed conditions up to the member age  $t$

$$\varepsilon'_{as}(t, t_s) = \gamma \cdot \kappa_{as} \cdot \varepsilon'_{ash\infty} [1 - \exp\{-\alpha_{as}(t - t_s)^{\beta_{as}}\}] \quad (5)$$

$$\varepsilon'_{ash\infty} = \frac{1170}{1 + 2.9e^{-0.019(50-W/C)}}$$

$$\alpha_{as} = 0.020 W/C - 0.75, \text{ where } \alpha_{as} \geq 0.1$$

$$\beta_{as} = -0.0085 W/C + 0.76$$

$$\kappa_{as} = 1 + \alpha_{as}(V/S - 25)$$

$$\alpha_{as} = 0.00014 W/C - 0.0051 \quad (35 \leq W/C \leq 45\%), \alpha_{as} = -0.0012 \quad (45 \leq W/C \leq 60\%)$$

Where  $\varepsilon'_{ash\infty}$  : final value of the shrinkage strain ( $\times 10^{-6}$ ) under sealed conditions of a  $100 \times 100 \times 400$  mm square prism,  $t$  : concrete member age (days),  $t_s$  : commencement of setting (days),  $\alpha_{as}$ ,  $\beta_{as}$  : coefficients representing the shrinkage progression properties under sealed conditions,  $\kappa_{as}$  : coefficient related to the final value of shrinkage under sealed conditions,  $\gamma$  : coefficient representing the effect of the type of cement and admixtures (may be taken to be 1.0 in the case of ordinary Portland cement only),  $W/C$  : water cement ratio (%),  $V/S$  : volume to surface area ratio (mm),  $RH$  : relative humidity (%).

Note that the range of application of Equations (4) and (5) are water cement ratio:  $35\% \leq W/C \leq 60\%$ , relative humidity:  $50\% \leq RH \leq 80\%$ , temperature:  $15^\circ\text{C} \leq T \leq 25^\circ\text{C}$ , volume surface area ratio:  $35\text{mm} \leq V/S \leq 300$  mm, and minimum member thickness:  $100 \text{ mm} \leq d_f$ .

### (iii) Method of calculating concrete creep

It is necessary to take into consideration the effect on concrete creep of the movement and dissipation of moisture in accordance with the structure. Normally it may be calculated from Equation (6).

$$\phi_t = \alpha \cdot \phi \quad (6)$$

Where  $\phi_t$  : creep coefficient used in calculating the long-term displacement,  $\phi$  : creep coefficient,  $\alpha$  : coefficient taking into consideration the long-term progression in accordance with the state of drying of the structure, may be 1.0 or higher.

### References

- 1) E. Witchkreangkrai, K. Tsuchida, T. Maeda, Y. Watanabe: Long-term deflection monitoring of a cantilever prestressed concrete bridge with intermediate hinges over 25 years. *Creep, Shrinkage and Durability Mechanics of Concrete and Concrete Structures: Proceedings of the CONCREEP 8 Conference Held in Ise-Shima, Japan, 30 September - 2 October 2008*, pp.595-600, CRC Press, 2008
- 2) K. Maekawa, T. Ishida, and T. Kishi: *Multi-Scale Modeling of Concrete*, Taylor and Francis, 2008
- 3) M. Ohno, N. Chijiwa, B. Suryanto, K. Maekawa: An Investigation into the Long-Term Excessive Deflection of PC Viaducts by Using 3D Multi-scale Integrated Analysis, *Journal of Advanced Concrete Technology*, Vol. 10, No. 2, pp. 47-58, 2012